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Passing through the 'chiral limit' in quenched QCD with Wilson fermions

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We investigate the limit of vanishing quark mass in quenched lattice QCD with unimproved Wilson fermions at $\beta = 6.0$. Exploiting the correlations of propagators at different time slices we extract pion masses extremely close to the 'chiral limit', despite the presence of 'exceptional configurations'. With this at hand, the existence of quenched chiral logarithms can be demonstrated, provided, finite size effects are small. With reference to the phase diagram proposed by Aoki [1] also the range $\kappa > \kappa_c$ is investigated. The width of a potential parity-flavor violating phase can, if at all, hardly be resolved.

1. INTRODUCTION

A recent investigation [2] pointed out that two basic scenarios are possible in the case of two-flavor QCD with Wilson fermions when the quark mass is tuned to zero in the thermodynamic limit:

(a) Aoki phase ($\mathbb{A}$)

– there exists a phase in which parity-flavor symmetry gets spontaneously broken at *finite* lattice spacing a – this phase is characterized by two corresponding massless Goldstone bosons $\pi^{\pm}$, whereas π^0 is massive, except on the phase boundaries,

– in the limit $a \to 0$ the pattern of parity-flavor symmetry breaking can be identified with the 'genuine' chiral symmetry breaking,

– for small a, the width of $\mathbb{A}$ should scale as $\Delta\frac{1}{\kappa} \sim a^3$ (up to log. corrections)

(b) there is no symmetry breaking involved in the 'chiral limit' at *finite* lattice spacing,

– correspondingly, this case would be indicated by equal, non vanishing pion masses in the limit $\kappa \to \kappa_c$; (but under certain conditions one can expect $m_\pi \sim O(a)$)

It is a priori not known, whether both scenarios can coexist in the phase diagram (as a function of β). Based on quenched numerical observation [3], the Aoki scenario is reported to hold at least up to $\beta \simeq 5.7$. In the full case [4], the existence of $\mathbb{A}$ was shown up to $\beta \sim 4$, the reported absence of $\mathbb{A}$ for $\beta \geq 5$ could also be interpreted as a sign for a narrow Aoki phase at larger β. For the finite temperature case, see [5].

In fact, all investigations of the low-lying spectrum are based on the *assumption*, that, at least in the thermodynamic limit, the pion mass(es) will eventually vanish at some value of κ in accordance with PCAC. This is surely true in the continuum limit. But in general, a behavior like $m_\pi^2 \propto m_q$ for small m_q does not automatically guarantee the existence of a *chiral limit* in a theory with matter fields realized by Wilson fermions, since other symmetries may take over the rôle of chiral symmetry at finite a. In this respect it is crucial to obtain direct information from the vicinity of $\kappa_c(\beta)$, as it is the aim of this study. In addition, we address the question of quenched chiral logs.

The model under consideration is standard Wilson lattice QCD with action $S = S_G + S_F$, S_G denoting the plaquette $SU(3)$ gauge part and

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S_F the fermionic action:

$$S_F = \sum_{x,y} \bar{\psi}_x \mathbb{M}_{xy} \psi_y \,, \quad \mathbb{M} = \hat{1} - \kappa \mathbb{D} \,,$$

$$\mathbb{D}_{xy} = \sum_{\mu} \left[\delta_{y,x+\hat{\mu}} P_\mu^- U_{x\mu} + \delta_{y,x-\hat{\mu}} P_\mu^+ U_{x-\hat{\mu},\mu}^\dagger \right] \quad (1)$$

where $P_\mu^\pm = \hat{1} \pm \gamma_\mu$. On each configuration, we use the following (non-singlet) correlator in order to extract the π mass :

$$\Gamma_\pi(\tau) \sim \sum_{\vec{x},\vec{y}} \mathrm{Tr} \left(\mathcal{M}_{xy}^{-1} \gamma_5 \mathcal{M}_{yx}^{-1} \gamma_5 \right) \qquad (2)$$

with $\tau = x_4 - y_4$. The current data sample consists of $O(100)$ quenched measurements each on $16^3 \times 32$ and $8^3 \times 32$ lattices at $\beta = 6.0$. As inversion methods, BiCGstabI (CG) have been used for $\kappa < \kappa_c$ ($\kappa > \kappa_c$).

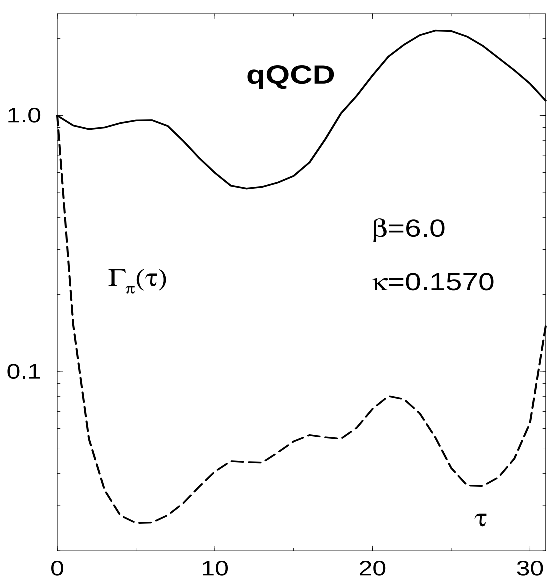

Figure 1. Typical 'exceptional' π propagators which have been part of the data sample.

2. THE VICINITY of $\kappa_c(\beta = 6.0)$

The quenched approximation is featured by 'exceptional configurations' appearing at small values of the quark mass. Typical examples of 'exceptional' π propagators which have been part of the data sample are given in Fig.1. It is common practice to discard them 'by hand' from the sample. But, viewed in a sufficiently large ensemble the individual Γ_π's (including 'exceptional' cases) obey a special form of linear correlation at different time slices. In this case we apply a variance reduction technique [6] taking advantage of $\langle x/y \rangle = \langle x \rangle \,/\, \langle y \rangle$, with $x = \Gamma_\pi(\tau+1)$, $y = \Gamma_\pi(\tau)$. Without reducing the data sample this allows to extract physical information in a region of extremely small m_q. For other approaches to the problem of 'exceptional configurations', see [7].

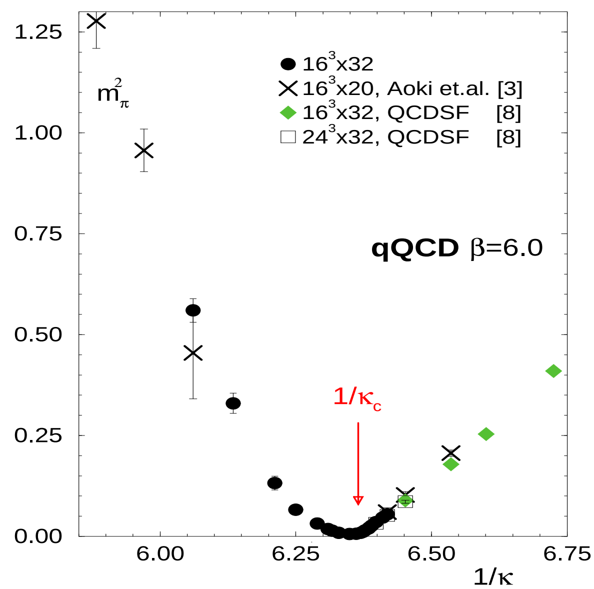

Figure 2. View of m_π^2 vs. $1/\kappa$ at $\beta = 6.0$.

In Fig.2 we display the behavior of m_π^2 vs. $1/\kappa$ at $\beta = 6.0$. This quantity represents the $(\text{mass})^2$ of the $\pi^\pm$, which, in full ($N_f = 2$) QCD, should vanish linearly in $1/\kappa$ for $\kappa \to \kappa_c$ and remain zero within $\mathbb{A}$. This behavior is predicted [2] to be symmetric with respect to the approach to the phase boundaries of $\mathbb{A}$ from outside. This is not the case as seen in Fig.2. Since the scenario depends on N_f, quenching effects might be large - there is need to quantify them here. At present, there is no theoretical handle to extrapolate $m_\pi^2 \to 0$ for the data at $\kappa > \kappa_c$. Even if one would enforce a linear dependence as has been done in [3] the modulus of the slope would be different from that found for $\kappa < \kappa_c$. Hence, the width of a potential Aoki phase cannot be firmly resolved at $\beta = 6.0$. This, however, is not in con-

tradiction with $\Delta\frac{1}{\kappa} \sim a^3$. For the minima of m_π^2 the existence of $\mathbb{A}$ would imply $m^2_{\pi,min} = 0$ for infinite volume. This is supported by a comparison with $8^3 \times 32$ data[2], but has to be checked on various lattice sizes.

3. QUENCHED CHIRAL LOGS

From the discussion in sect.2 it is justified to assume the presence of quenched *chiral* logarithms in the data below κ_c. Based on quenched chiral perturbation theory one expects [9] :

$$\ln\left(\frac{m_\pi^2}{m_q}\right) = c_0 - \frac{\delta}{\delta+1}\ln m_q + c_1 m_q + c_2 m_q^2 \quad (3)$$

with $\delta = m_0^2/(24\pi^2 f_\pi^2) > 0$. Fig.3 represents $\ln(m_\pi^2/m_q)$ as a function of m_q. For very small m_q the term $\propto \ln m_q$ should dominate, which is seen in our data points for $\kappa < \kappa_c$. The result for δ sensitively depends on the determination of m_q. It is more reasonable to eliminate κ_c entering eq.(3) as a free parameter by using directly the PCAC quark mass, if possible (cf. [10]). Preliminary, we fixed $\kappa_c (= 0.15693)$ by a χ^2 fit to our 7 data points in the range indicated by the the vertical lines in Fig.3 assuming $c_1 = c_2 = 0$ and obtaining $\delta \sim 0.3$ as a first estimate. The data points at larger values of κ have been omitted, because of expected finite size effects.

We can draw the qualitative conclusion, that our data is compatible with quenched chiral logarithms, with a value of δ in the expected range. However, it requires more investigation in order to come to a quantitative conclusion.

4. SUMMARY

We conclude, that at the given β value a finite width of the Aoki phase cannot be firmly resolved. At the same time, quenched *chiral* logarithms are visible in our data.

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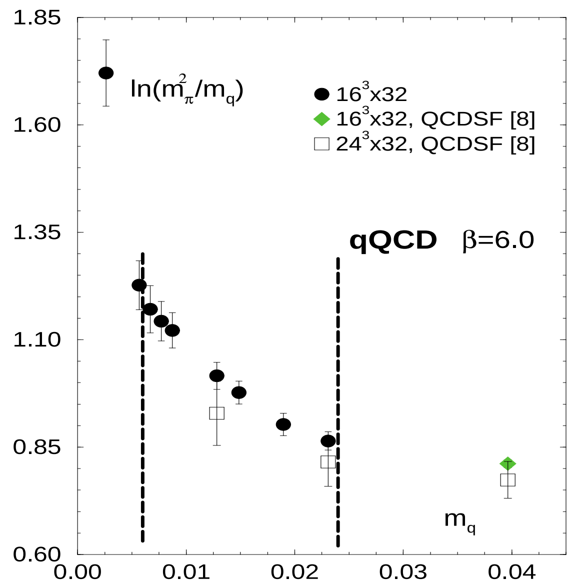

Figure 3. $\ln(m_\pi^2/m_q)$ as a function of m_q.

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